Total number of printed pages–11

2022

STATISTICS

(Honours)

Paper : STA–HC–1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : ***(any ten)*** 1×10=10

(a) If a function is derivable at all points of an interval except the ends points, it is said to be derivable in the open interval. *(State True **or** False)*

(b) The value of the integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx \text{ is}$$

(i) $\beta(m,n)$

Contd.

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(ii) $\beta(n,m)$

(iii) Both (i) and (ii)

(iv) None of the above

(Choose the incorrect option)

(c) The *n*th derivative of a^x is

(i) a^x

(ii) $(\log_e a)^n a^x$

(iii) na^x

(iv) None of the above

(Choose the correct option)

(d) Evaluate $\int_0^\infty e^{-3x} x^{\frac{1}{2}} dx$

(e) The differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 - 2\left(\frac{dy}{dx}\right)^2 + 5y = 0$ is of order _____ and degree ______.

(Fill in the blanks)

(f) State *two* properties of definite integrals.

3

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(g) Define homogeneous function of two variables.

(h) If $f(x,y) = x^4 + xy + y^4$ find f_x and f_{yx}.

(i) The value of

$$\lim_{x \to \infty} \frac{x^2}{e^x} \text{ is}$$

(i) ∞

(ii) 0

(iii) $\frac{\infty}{\infty}$

(iv) None of the above

(Choose the correct option)

(j) Write *two* properties of double integrals.

(k) The differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy \text{ is called}$$

(i) ordinary differential equation

(ii) partial differential equation

(iii) None of the above

(Choose the correct option)

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(l) Find the differential equation of all the straight lines passing through the origin.

(m) Define beta integral of second kind.

(n) Lagrange's undetermined multipliers is a method of finding the ______ or ______ of a function subject to one or more conditions. *(Fill in the blanks)*

(o) Define Jcobian of the functions $u_1, u_2 \dots u_n$ with respect to $x_1, x_2 \dots x_n$.

(p) The value of $\Gamma(n+1)$ is

(i) $n!$

(ii) $n\Gamma(n)$

(iii) Both (i) and (ii)

(iv) $(n-1)!$

(Choose the incorrect option)

(q) The function x^n is continuous for all values of x when n is positive and continuous for all values of x except 0 when n is negative.

*(State True **or** False)*

(r) Define bounded function.

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2. Answer ***any five*** of the following questions : 2×5=10

(a) Test the differentiability of the function

$$f(x) = \begin{cases} 1+x, & \text{if } x \le 2 \\ 5-x, & \text{if } x > 2 \end{cases}$$

at $x = 2$

(b) Find the *n*th differential coefficients of $\sin^3 x$.

(c) If $f(x) = x.\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$, $x \neq 0$ and $f(0) = 0$,

show that $f(x)$ is continuous at $x = 0$.

(d) Find the value of

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

(e) Show that $f(x) = 2x^3 - 21x^2 + 36x - 20$ has a maximum at $x = 1$.

(f) Prove that

$$\beta(m,n) = \beta(m+1, n) + \beta(m, n+1)$$

(g) Solve the

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$$

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(h) Prove that

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$$

(i) If $x^3 + 3x^2y + 6xy^2 + y^3 = 1$, find $\frac{dy}{dx}$.

(j) If u be a homogeneous function of x and y of degree n, then show that

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$$

3. Answer ***any four*** from the following questions : 5×4=20

(a) Show that the function $f(x) = |x| + |x-1|$ is not differentiable at $x = 1$ but differentiable at $n = 2$.

(b) If $y = e^{a \sin^{-1} x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$$

(c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

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(d) Find the solution

$$\lim_{x\to 0} \frac{\tan x - x}{x - \sin x}$$

(e) If f_x and f_y are both differentiable at a point (a,b) of domain of definition of a function f, then

$$f_{xy}(a,b) = f_{yx}(a,b)$$

(f) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$$

(g) If $0 < n < 1$, then show that

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

Hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(h) Evaluate

$$\int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y)\,dy\,dx$$

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4. Answer ***any four*** from the following questions : 10×4=40

(a) (i) Prove that

$$\int_{-a}^{a} f(x)dx = 0, \text{ if } f(x) \text{ is and odd function of } x.$$

$$= 2\int_{0}^{a} f(x)dx, \text{ if } f(x) \text{ is an even function of } (x).$$ 3

(ii) Using properties of definite integral prove that

$$\int_{0}^{\pi/2} \log \sin x \, dx = \frac{\pi}{2} \log \frac{1}{2}$$ 7

(b) (i) Find the differential coefficient of $x^x + (\sin x)^{\log x}$ 5

(ii) If $y = e^x \log x$, show that in usual notation

$$xy_2 - (2x-1)y_1 + (x-1)y = 0$$ 5

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(c) (i) If $f(x,y) = \frac{2xy(x^2 - y^2)}{x^2 + y^2}$,

$(x,y) \neq (0,0)$

$f(0,0) = 0$, find $f_x(0,0)$ and $f_y(0,0)$ 5

(ii) If $u = \log\left\{\frac{x^2 + y^2}{x + y}\right\}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$ 5

(d) (i) Explain general solution of Clairaut's equation. 4

(ii) Solve the equation $(px - y)(x - yp) = 2p$ to Clairaut's form by the substitution $x^2 = u$, $y^2 = v$, and find its solution. 6

(e) (i) Show that

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$$ 6

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(ii) Prove that

$$\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$$ 4

(f) (i) Explain the procedure of equations solvable for p and y. 5

(ii) Solve the differential equation

$$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - 56 = 0$$ 5

(g) (i) State the necessary and sufficient condition for extreme value of a function of two variables. 4

(ii) Find the maximum value of

$$f(x, y) = 3x^2 - y^2 + x^3$$ 6

(h) Prove that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} \, dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} \, dx$$

(i) The roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ are u, v, w. Prove that

$$\frac{\partial(u\ v\ w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

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(j) (i) Define partial differential equation. 1

(ii) Solve the partial differential equations

(A) $\left(\frac{y^2 z}{x}\right)p + xzq = y^2$ 5

(B) $xp + yq = z$ 4

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