3 (Sem–4/CBCS) PHY HC 1

2021

PHYSICS

(Honours)

Paper : PHY–HC–4016

(Mathematical Physics – III)

Full Marks : 60

GROUP–A

(Full Marks : 30)

1. Answer the following questions : 1×5= 5

(a) Write the polar form of a complex number.

(b) State Convolution theorem on Fourier transform.

Contd.

(c) Prove $L(1)=\frac{1}{s}$, $s>0$, using the definition of Laplace transform.

(d) Find the Residue of the complex function $f(z)=\frac{1}{1+z^2}$ at the pole $z=i$.

(e) Write the law of transformation for the tensor A_n^{lm}.

2. Answer the following questions : 2×5=10

(a) Find the modulus and argument of $-3i$.

(b) Check whether the function $f(Z) = ReZ$ is analytic or not.

(c) Write the Fourier's sine and cosine transform.

(d) If $L[f(x)] = \bar{f}(s)$, then show that $L\left[e^{ax} f(x)\right] = \bar{f}(s-a)$.

(e) Prove that a symmetric tensor of rank 2 in N dimensional space has $\frac{N(N+1)}{2}$ independent elements.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Prove the Cauchy's integral formula —

$$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$$

(b) Using Cauchy's residue theorem, show that —

$$\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$$

(c) Find the Fourier transform of the function —

$$f(x)=\begin{cases}1, & \text{for } |x|<a\\ 0, & \text{for } |x|>a\end{cases}$$

(d) Find the Laplace transform of $f(t)$, where,

$$f(t)=\begin{cases}\sin t, & 0<t<\pi\\ 0, & t>\pi\end{cases}$$

(e) What is Kronecker delta ? Show that it is a mixed tensor of rank 2.

GROUP–B

(Full Marks : 30)

4. Answer ***any three*** of the following questions :

(a) (i) Obtain the Cauchy-Reimann conditions for the function $f(z)=u+iv$ to be analytic, in a domain, u and v are functions of x and y. Are the conditions sufficient ? 6+1 =7

(ii) Expand the following function in Taylor series :

$$f(z)=\frac{1}{z+1}, \text{ about } z=1. \qquad 3$$

(b) Evaluate the following integrals using calculus of residues : $5\times2=10$

(i) $\int_{-\infty}^{+\infty} \frac{\sin x}{x} dx$

(ii) $\int_{0}^{2\pi} \frac{d\theta}{5-4\sin\theta}$

(c) Using Fourier transform, solve the one-dimensional heat flow equation

$$\frac{\partial u(x,t)}{\partial t}=\frac{\partial^2 u(x,t)}{\partial x^2}, \; x>0, \; t>0, \text{ subject to}$$

the conditions –

(i) $u(0,t)=0$

(ii) $u(x,0)=\begin{cases}1, & 0<x<1\\ 0, & x\geq 1\end{cases}$

(iii) $u(x,t)$ is bounded. 10

(d) Evaluate the following inverse Laplace transforms – 4+6=10

(i) $L^{-1}\left[\frac{s^2}{(s+1)^2}\right]$

(ii) $L^{-1}\left[\frac{4P+5}{(P-4)^2(P+3)}\right]$

(e) (i) What is Levi-Civita tensor ? Prove that $\delta_{ij} \in_{ijk} = 0$. 2+1=3

(ii) Prove that

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^{3} \sum_{j=1}^{3} A_i B_j \delta_{ij} .$$ 3

(iii) Prove that tensor of rank 2 could be written as a sum of symmetric and asymmetric tensor. 4